# 4.4 Rotation Matrices in 2-Dimensions

## The Rotation Matrix

To this point, we worked with vectors and with matrices. Now, we will put them together to see how to use a matrix multiplication to rotate a vector in the counterclockwise direction through some angle in 2-dimensions.

|  |  |
| --- | --- |
| Diagram of a vector v in a coordinate system and its counterclockwise rotation through some angle. | Diagram of a vector v and its rotational  vector v prime. Vector v is rotated counterclockwise through an angle and v prime is new vector after the rotation. |

Our plan is to rotate the vector counterclockwise through some angle to the new position given by the vector . To do so, we use the rotation matrix, a matrix that rotates points in the -plane counterclockwise through an angle relative to the -axis.

## The Rotation Process

To get the coordinates of the new vector perform the matrix multiplication

Find the vector that results when the vector is rotated 90° counterclockwise.

Example (1)

Using the rotation formula with

and we get

When rotated counterclockwise 90°, the vector becomes .

|  |  |
| --- | --- |
| Diagram showing vector (1,-1). | Diagram showing vector (1,-1) and a new vector (1,1) after vector (1,-1) is rotated counterclockwise through 90 degrees. |

Find the vector that results when the vector is rotated 60° counterclockwise.

Example (2)

Using the rotation formula with and we get

When rotated counterclockwise 60°, the vector becomes .

|  |  |
| --- | --- |
| Diagram showing vector (2,6). | Diagram showing vector (2,6) and a new vector (-4.20,4.73) after vector (2,6) is rotated counterclockwise 60 degrees. |

## Using Technology

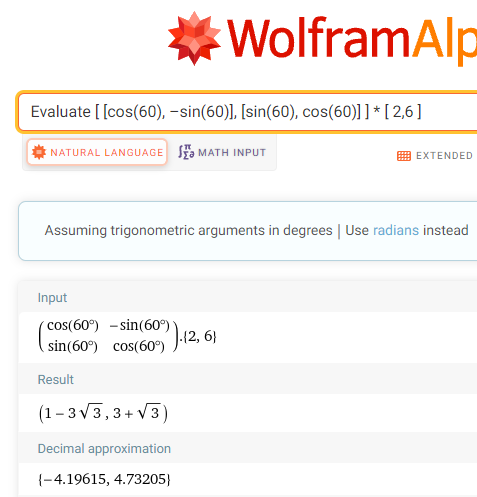
We can use technology to help us find the rotation. WolframAlpha evaluates the trig functions for us.

Go to www.wolframalpha.com.

We can check the above problem from Example 2 by using WolframAlpha. Find the vector that results when the vector is rotated 60° counterclockwise.

To find rotation of the vector enter Evaluate [ [cos(60), –sin(60)], [sin(60), cos(60)] ] \* [ 2,6 ] into the entry field. Both entries and rows are separated by commas and W|A does not see spaces.

WolframAlpha tells you what it thinks you entered, then it shows you the answer.



When rotated counterclockwise 60°, the vector becomes

## EXAMPLES

1. through 90°. ANS:
2. through 180°. ANS:
3. through 270°. ANS:
4. through 90°. ANS:
5. through 45°. ANS:
6. through 45°. ANS:
7. through °. ANS:
8. through °. ANS:
9. Approximate, to five decimal places, the coordinates of the vector when it is rotated counterclockwise 30°.

ANS:

## Note to Instructor

Note that we plan to rotate some vector through some angle to the new position given by the vector , and to do so, we will use the rotation matrix, a matrix that rotates points in the -plane counterclockwise through an angle relative to the -axis.

Consider demonstrating these rotations:

1. Find the vector that results when the vector is rotated 90° counterclockwise.

Using the rotation formula with and we get

When rotated counterclockwise 90°, the vector becomes .

* If your class knows some trig, you can show the conversion of

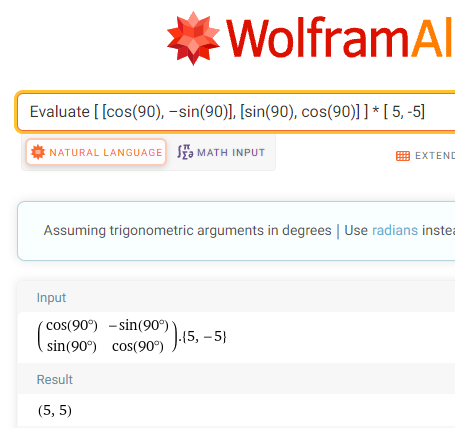
to

Since

* If trig is a challenge, use WolframAlpha to perform the matrix multiplication.

Go to www.wolframalpha.com.

To find rotation of the vector, enter Evaluate [ [cos(90), –sin(90)], [sin(90), cos(90)] ] \* [ 5, -5] into the entry field. WolframAlpha tells you what it thinks you entered, then tells you its answer.

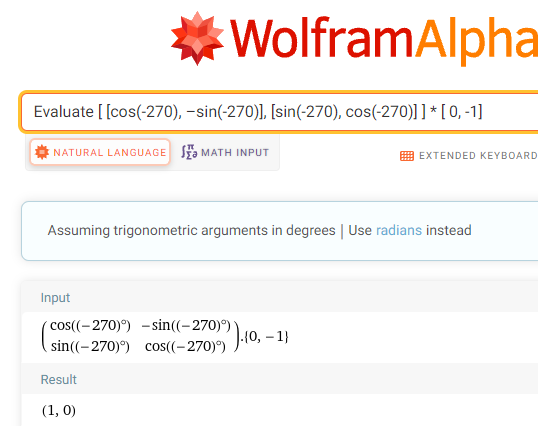


Be sure to write a conclusion so your students know to do so.

When rotated counterclockwise 90°, the vector becomes .

1. The rotation formula works for clockwise rotations. We just need to make the angle of rotation negative.

Find the vector that results when the vector is rotated –270°.

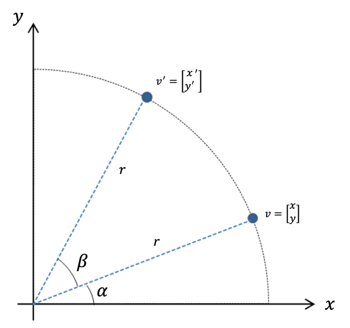


When rotated clockwise 90°, the vector becomes .

## Deriving the Rotation Formula

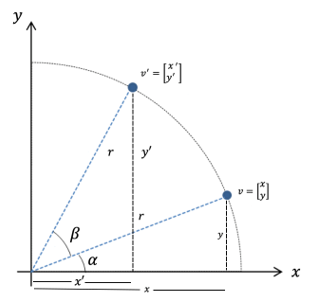
If your class knows some trig, you may wish to derive the rotation formula.

We wish to derive a formula that rotates a vector counterclockwise through some angle to the new position given by the vector .



We wish to rotate the vector through an angle around the origin.

We know that in general,



The figure shows that for angle ,

Also

By the trigonometry addition identity,

Then, since and

Replace with and with

Similarly, by the trigonometry addition identity,

Then, since and

Replace with and with

Rewrite this as

Now we have

Putting these two results into matrix form, .

Replacing with to match our notation, we get

And we have produced the rotation formula.

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